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MATHEMATICS
HIGHER LEVEL
PAPER 3 – SERIES AND DIFFERENTIAL EQUATIONS

Tuesday 19 November 2013 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[60 marks]*.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 10]*

Consider the infinite series $\sum_{n=1}^{\infty} \frac{2}{n^2+3n}$.

(a) Use a comparison test to show that the series converges. *[2]*

(b) (i) Express $\frac{2}{n^2+3n}$ in partial fractions.

(ii) Hence find the value of $\sum_{n=1}^{\infty} \frac{2}{n^2+3n}$. *[8]*

2. *[Maximum mark: 9]*

The general term of a sequence $\{a_n\}$ is given by the formula $a_n = \frac{e^n + 2^n}{2e^n}$, $n \in \mathbb{Z}^+$.

(a) Determine whether the sequence $\{a_n\}$ is decreasing or increasing. *[3]*

(b) Show that the sequence $\{a_n\}$ is convergent and find the limit L. *[2]*

(c) Find the smallest value of $N \in \mathbb{Z}^+$ such that $|a_n - L| < 0.001$, for all $n \geq N$. *[4]*

3. *[Maximum mark: 19]*

Consider the differential equation $\frac{dy}{dx} = \frac{y}{x + \sqrt{xy}}$, for $x,\ y > 0$.

(a) Use Euler's method starting at the point $(x,\ y) = (1,\ 2)$, with interval $h = 0.2$, to find an approximate value of y when $x = 1.6$. *[7]*

(b) Use the substitution $y = vx$ to show that $x\frac{dv}{dx} = \frac{v}{1+\sqrt{v}} - v$. *[3]*

(c) (i) Hence find the solution of the differential equation in the form $f(x,\ y) = 0$, given that $y = 2$ when $x = 1$.

(ii) Find the value of y when $x = 1.6$. *[9]*

4. *[Maximum mark: 13]*

Let $g(x) = \sin x^2$, where $x \in \mathbb{R}$.

(a) Using the result $\lim\limits_{t \to 0} \frac{\sin t}{t} = 1$, or otherwise, calculate $\lim\limits_{x \to 0} \frac{g(2x) - g(3x)}{4x^2}$. *[4]*

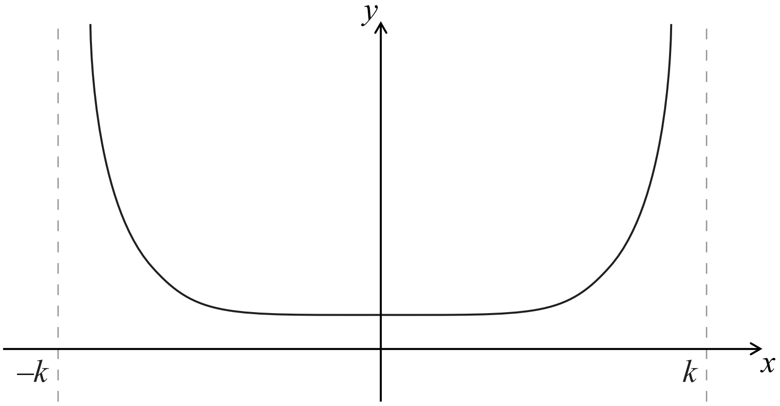
(b) Use the Maclaurin series of $\sin x$ to show that $g(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{(2n+1)!}$. *[2]*

(c) Hence determine the minimum number of terms of the expansion of $g(x)$ required to approximate the value of $\int_0^1 g(x)\,\mathrm{d}x$ to four decimal places. *[7]*

5. *[Maximum mark: 9]*

A function f is defined in the interval $]-k, k[$, where $k > 0$. The gradient function f' exists at each point of the domain of f.

The following diagram shows the graph of $y = f(x)$, its asymptotes and its vertical symmetry axis.



(a) Sketch the graph of $y = f'(x)$. *[2]*

Let $p(x) = a + bx + cx^2 + dx^3 + \ldots$ be the Maclaurin expansion of $f(x)$.

(b) (i) Justify that $a > 0$.

(ii) Write down a condition for the largest set of possible values for each of the parameters b, c and d. *[5]*

(c) State, with a reason, an upper bound for the radius of convergence. *[2]*